

Reading guide – Berard part 2

Correction needed to presented proofs for Berard’s Theorem 3 and Lemma 4! As was pointed out in class, if $M = A \cap B$, then \overline{M} need not equal $\overline{A} \cap \overline{B}$. However, it is true that $\overline{M} \subseteq \overline{A} \cap \overline{B}$. This containment is all we need: We showed a set that *contains* the boundary of M is empty, hence the boundary of M is also empty.

Berard Lemma 8.

Exercise: Show that if $x > m$, a y can be found such that $y < m < x$ and $x, y \in B(m, \epsilon)$. This is similar to the proof giving the existence of the desired y when $x < m$ but involves more than just changing notation.

Berard Theorem 9.

Theorem (a) on page 2 of the lecture notes is used. There are three cases, depending on whether m is a cut point, the “chosen” non-cut point, or a second non-cut point. There is a flaw in Berard’s proof in the third case, as the exercise below demonstrates. (We’ll fix the flaw to the proof in class.)

Exercise: Let $X = \{(x, y) : (x - 1)^2 + y^2 = 1, 0 \leq y \leq 1\} \subset \mathbf{R}^2$ and let d be the metric on X induced by the usual Euclidean metric on \mathbf{R}^2 . There are two non-cut points in X : let $z = (0, 0)$ and $m = (2, 0)$. Choose $\epsilon \in (0, 1/4]$. Show that if x is the unique point in X such that $d(z, x) = d(z, m) - \epsilon/4$, then $(x, +\infty) \not\subseteq B(m, \epsilon)$, contrary to what Berard claimed.

Specifically, show $d(m, x) - \epsilon > 0$. (This requires facts about distances in \mathbf{R}^2 and the triangle inequality.) Then, since x is strictly more than ϵ away from m , a y can be found such that $x < y \notin B(m, \epsilon)$ (Use Lemma 8 or another method). Thus, $(x, +\infty) \not\subseteq B(m, \epsilon)$.

(This flaw was corrected in *Unique Midpoint Property in Metrizable Spaces: A Survey* by Yuri Kitai.)

Berard Theorem 10.

Definition: A *subbasis* \mathcal{S} for a topology on X is a collection of subsets of X whose union equals X . The topology generated by the subbasis \mathcal{S} is defined to be the collection τ of all unions of finite intersections of elements of \mathcal{S} . (Compare this definition to that of “basis.”)

Definition: An *open* function is a function that maps open sets to open sets. If a function f has an inverse, then saying f is open is equivalent to saying f^{-1} is continuous.

Note that f is shown to be open by showing f takes a subbasis of X to a collection of sets in $f(X)$ that is a subbasis that generates the usual topology on subsets of \mathbf{R} .

Berard Theorem 12.

Exercise: Prove that $A \cup \{z\}$ is connected. (Although Berard cites Moore, this is not hard. The Moore text is in the math library if you want to look up the proof, but it will take you as long to decipher Moore’s notation as to write your own proof.)