On Monday, we will continue working on Lemma 2 in the Ohta/Ono paper – which means working on the Berard paper. Lectures tend to make more sense when you have at least a general idea of the material. Browse the Berard paper. The following are items that may be helpful to think about as you read.

Berard Lemma 1. Why are the sets A and B open?

Berard Lemma 2.
Rewrite this lemma in an equivalent form using the phrase "z is a a cut point of X". There are, of course several ways to do this. Can you work the term "midset" in?

Berard Theorem 3.
Recall notation: $B(z, \epsilon) = \{q \in X : d(z, q) < \epsilon\}$
$\overline{B}(z, \epsilon) = \{q \in X : d(z, q) \leq \epsilon\}$
$c\overline{B}(z, \epsilon) = \{q \in X : d(z, q) > \epsilon\}$
$cB(z, \epsilon) = \{q \in X : d(z, q) \geq \epsilon\}$
$D(z, \epsilon) = \{q \in X : d(z, q) = \epsilon\}$

In the proof of Theorem 3, the set M is defined: $M = B(z_2, \epsilon_2) \cap cB(z_3, \epsilon_1)$. Write down an expression for the set $Bd(M)$. If $y \in Bd(M)$, what does that say about the distance between y and z_2 or z_3?

Definition of $<$ and Lemmas 5 and 6.
Recall: A relation $<$ on a set X is called a *simple order* if it has the following properties:

- (comparability) For every x and y in X for which $x \neq y$, either $x < y$ or $y < x$.
- (nonreflexivity) For no $x \in X$ does the relation $x < x$ hold.
- (transitivity) If $x < y$ and $y < z$, then $x < z$.

The proofs of these lemmas are straight-forward. Give them a try. The "weaker" topology has fewer open sets (that is, is "coarser").

Berard Lemma 7.
Why, if $I(a, b)$ is not connected, is it the union of two nonempty, disjoint, CLOSED sets?