Analysis -- Paper Assignment 1

1. Read the Ohta/Ono paper. Make a list of topics you think we'll need to learn in order to understand the paper well. What words need to be defined? Please turn this in on Tuesday.

2. Use the theorems in the Ohta/Ono paper to decide whether the following subspaces of **R** definitely do have UMP, definitely don't have UMP, or to which the theorems do not apply. You do not need to find a metric with UMP or be able to find midpoints, but do cite which theorem(s) apply. Try this individually. (This is an exercise in applying theorems – and depending on your background, you may not understand the theorems. Don't worry! By the end of IMMERSE, this will be much easier.)

$[0, 1] \cup \{2\}$	$[0, 1] \cup [2, 3] \cup [4, 5]$	$\bigcup_{n=1}^{5}[n, n+1/2]$
$\bigcup_{n=1}^{8}\{n\}$	$[0, 1] \cup \mathbf{Q}$	$\bigcup_{n=1}^{14}[n, n+1/2]$
$[0, 1] \cup [2, 3]$	$(0, 1) \cup \mathbf{Q}$	$\mathbf{R} - \mathbf{Q}$
$[0, 1) \cup (2, 3)$	$\bigcup_{n=1}^{4}[n, n+1/2]$	The Cantor Set
$\{1\} \cup \{2\} \cup \{3\}$	$\{0\} \cup \{1\}$	$\bigcup_{n=1}^{\infty}[n, n+1/2]$

3. Let $X = \{(-1,1)\} \cup \{(x, x^2) : x \geq 0\}$.

   a. Verify that *X* with the usual metric in $\mathbf{R}^2$ has UMP.

   b. Find the midpoint of various specific pairs of points in *X*.

   c. Find a formula for the medial map *m(x, y)* for distinct *x, y* in *X*.

   d. Whether or not you were successful at (c), what can you discover about the medial map? For example, if *m(a, b) = m(x, y)*, what can be said about points *a, b, x,* and *y*? Or, fix one point and explore limits as the second point moves. Is *m* continuous? Is it onto?

4. Let $Y = \{-1\} \cup [0, \infty)$. The usual metric on *Y* does not have UMP, but *Y* has UMP. Why? Can you define a metric on *Y* that does have UMP?