1. Definition and Exploration of UMP

A metric d on a nontrivial space X is said to have the *unique midpoint property* (UMP) if for each pair of distinct points x, y in X there exists one and only one point $p \in X$ such that $d(x, p) = d(y, p)$.

The set $\{q \in X : d(x,q) = d(y,q)\}$ is called the *midset* of x and y.

When a metric has UMP, and p is the unique element in the midset of x and y, then we write $p = m(x, y)$, where m is the medial map from $X^2 \setminus \Delta \to X$. $(\Delta = \{(x, x) : x \in X\})$

1. Make sense of the definitions above and verify that the real numbers ***R*** with the usual metric has UMP.

2. Does $S = \{(x, y) \in R^2 : x^2 + y^2 = 1\}$ with the usual metric (or the shortest arc metric) have UMP?

3. Generate examples and non-examples of spaces with metrics with UMP.

4. A set *X* can have more than one metric defined on it. Is it possible that one metric has UMP and another doesn't? If so, give example(s). If not, why not?

There is a subtle distinction between saying a metric has UMP and a space has UMP. We will say a space has UMP if there is a topology preserving metric defined on it which has UMP.

5. Look at the examples we've generated. What qualities do spaces with UMP have? Consider qualities like open, closed, connected, compact, cardinality, etc.

6. What questions might a mathematician explore in regards to spaces with UMP?